

# Structural Typing for Structured Products

Tim Williams

Peter Marks

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# Background

## The FPF Framework

- A standardized representation for describing payoffs
- A common suite of tools for trades which use this representation
  - UI for providing trade parameters
  - Mathematical document descriptions
  - Pricing and risk management
  - Barrier analysis
  - Payments and other lifecycle events

## FPF Lucid

- A DSL for describing exotic payoffs and strategies
- Control constructs based around schedules
- Produces abstract syntax—allowing multiple interpretations
- Damas-Hindley-Milner type inference with constraints and polymorphic extensible row types

# Lucid language

Articulation driven design

## Lucid language

Articulation driven design

## Lucid type system

Structural typing with  
Row Polymorphism

A simple numeric expression

$\exp(x)$

A simple numeric expression

`exp(x)`

Monomorphic

`exp : Double → Double`  
`x : Double`  
`exp(x) : Double`

## A conditional expression

```
if c then x else y
```

## A conditional expression

**if** c **then** x **else** y

## Polymorphic

if\_then\_else\_ : (Bool, a, a) → a  
c : Bool  
x : a  
y : a  
if c then x else y : a

- Hindley-Milner type system

## Overloaded numeric literal

x + 42

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x + 42

## Subtyping

(+) : (Num, Num) → Num

42 : Integer

x : Num

x + 42 : Num

- Subtyping constraints difficult to solve with full inference
- A complex extension to Hindley-Milner

## Overloaded numeric literal

x + 42

## Polymorphic with type variable constraints

(+) : Num a  $\Rightarrow$  (a, a)  $\rightarrow$  a

42 : Num a  $\Rightarrow$  a

x : Num a  $\Rightarrow$  a

x + 42 : Num a  $\Rightarrow$  a

- Any type variable can have a single constraint
- Unifier ensures constraints are met
- Simple extension to Hindley-Milner

## A simple Lucid function

```
function capFloor(perf, cap, floor)
    return max(floor, min(cap, perf))
end
```

## A simple Lucid function

```
function capFloor(perf, cap, floor)           capFloor : Num a ⇒ (a, a, a) → a
    return max(floor, min(cap, perf))
end
```

## A simple Lucid function

```
function capFloor(perf, cap, floor)
    return max(floor, min(cap, perf))
end
```

```
capFloor(perf, 0, 1)
```

$\text{capFloor} : \text{Num } a \Rightarrow (a, a, a) \rightarrow a$

- Not obvious which argument when applying function

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
```

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
```

## Records via Nominal typing

```
data Num a ⇒ CapFloor a = CapFloor  
    {cap : a, floor : a }
```

```
capFloor : Num a ⇒ (a, CapFloor a) → a
```

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
```

## Records via Nominal typing

data Num  $a \Rightarrow$  CapFloor  $a = \text{CapFloor}$   
 $\{\text{cap} : a, \text{floor} : a\}$

capFloor : Num  $a \Rightarrow (a, \text{CapFloor } a) \rightarrow a$

- Don't want to force users to define data types

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
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## Records via Nominal typing

data Num  $a \Rightarrow$  CapFloor  $a = \text{CapFloor}$   
 $\{\text{cap} : a, \text{floor} : a\}$

capFloor : Num  $a \Rightarrow (a, \text{CapFloor } a) \rightarrow a$

- Don't want to force users to define data types
- Don't want to force users to name a combination of fields

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
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## Records via Nominal typing

data Num  $a \Rightarrow$  CapFloor  $a = \text{CapFloor}$   
 $\{cap : a, floor : a\}$

capFloor : Num  $a \Rightarrow (a, \text{CapFloor } a) \rightarrow a$

- Don't want to force users to define data types
- Don't want to force users to name a combination of fields
- Want to use the same fields in different data types

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
```

## Structural record types

capFloor : Num  $a \Rightarrow$   
 $(a, \{cap : a, floor : a\}) \rightarrow a$

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})  
    return max(floor, min(cap, perf))  
end
```

```
capFloor(perf, {cap=0, floor=1})  
capFloor(perf, {floor=1, cap=0})
```

## Structural record types

$$\begin{aligned} \text{capFloor} : \text{Num } a \Rightarrow \\ (a, \{ \text{cap} : a, \text{floor} : a \}) \rightarrow a \end{aligned}$$

- Unifier is agnostic to field order

## Grouping and labelling arguments

```
function capFloor(perf, {cap, floor})
    return max(floor, min(cap, perf))
end
```

```
capFloor(perf, {cap=0, floor=1})
capFloor(perf, {floor=1, cap=0})
```

## Structural record types

$\text{capFloor} : \text{Num } a \Rightarrow$   
 $(a, \{\text{cap} : a, \text{floor} : a\}) \rightarrow a$

- Unifier is agnostic to field order
- Note the above is still not quite what Lucid infers

## Grouping and labelling arguments

```
function capFloor(perf, r)
    return max(floor, min(r.cap, r.perf))
end
```

```
capFloor(perf, {cap=0, floor=1})
capFloor(perf, {floor=1, cap=0})
```

## Structural record types

capFloor : Num  $a \Rightarrow$   
 $(a, \{cap : a, floor : a\}) \rightarrow a$

- Unifier is agnostic to field order
- Note the above is still not quite what Lucid infers
- Pattern matching is just syntactic sugar for field selection

## Ignoring additional fields

```
function kgcf(perf, r)
    return capFloor( r.part * (perf - r.strike)
                    , {cap = r.cap, floor = r.floor})
end

kgcf(perf, {part=1, strike=0.9, cap=0, floor=1.2})
```

## Ignoring additional fields

```
function kgcf(perf, r)
    return capFloor(r.part * (perf - r.strike), r)
end

kgcf(perf, {part=1, strike=0.9, cap=0, floor=1.2})
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## Ignoring additional fields

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## Structural Record types

$\text{capFloor} : \text{Num } a \Rightarrow (a, \{\text{cap} : a, \text{floor} : a\}) \rightarrow a$

$\text{kgcf} : \text{Num } a \Rightarrow (a, \{\text{part} : a, \text{strike} : a, \text{cap} : a, \text{floor} : a\}) \rightarrow a$

- How do we allow a superset of fields to be passed to CapFloor?

## Ignoring additional fields

```
function kgcf(perf, r)
    return capFloor(r.part * (perf - r.strike), r)
end
```

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kgcf(perf, {part=1, strike=0.9, cap=0, floor=1.2})
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## Structural Record types

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- How do we allow a superset of fields to be passed to CapFloor?
- Subtyping would require a new type system and inference algorithm

## Ignoring additional fields

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function kgcf(perf, r)
    return capFloor(r.part * (perf - r.strike), r)
end
```

```
kgcf(perf, {part=1, strike=0.9, cap=0, floor=1.2})
```

## Polymorphic extensible Records

$\text{capFloor} : \text{Num } a \Rightarrow (a, \{\text{cap} : a, \text{floor} : a \mid r\}) \rightarrow a$

$\text{kgcf} : \text{Num } a \Rightarrow (\text{perf}, \{\text{part} : a, \text{strike} : a, \text{cap} : a, \text{floor} : a \mid s\}) \rightarrow a$

- How do we allow a superset of fields to be passed to CapFloor?
- Subtyping would require a new type system and inference algorithm
- Can use parametric polymorphism by using a type variable to represent the remaining fields

## Extending Records

```
function gcfBasket(perf, weights, r)
    return kgcf( sumProduct(perfs, weights)
                 , {strike=1, part=r.part, cap=r.cap, floor=r.floor})
end
```

## Extending Records

```
function gcfBasket(perf, weights, r)
    return kgcf(sumProduct(perfs, weights), {strike=1 |r})
end
```

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```
function gcfBasket(perf, weights, r)
    return kgcf(sumProduct(perfs, weights), {strike=1 |r})
end
```

## Polymorphic extensible Records

$\text{kgcf} : (\text{Num } a, s/\text{part/strike/cap/floor}) \Rightarrow$   
 $(a, \{\text{part} : a, \text{strike} : a, \text{cap} : a, \text{floor} : a \mid s\}) \rightarrow a$

$\text{gcfBasket} : (\text{Num } a, r/\text{part/strike/cap/floor}) \Rightarrow$   
 $(a, \{\text{part} : a, \text{cap} : a, \text{floor} : a \mid r\}) \rightarrow a$

## Extending Records

```
function gcfBasket(perf, weights, r)
    return kgcf(sumProduct(perfs, weights), {strike=1 |r})
end
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## Polymorphic extensible Records

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 $(a, \{\text{part} : a, \text{strike} : a, \text{cap} : a, \text{floor} : a \mid s\}) \rightarrow a$

$\text{gcfBasket} : (\text{Num } a, r/\text{part/strike/cap/floor}) \Rightarrow$   
 $(a, \{\text{part} : a, \text{cap} : a, \text{floor} : a \mid r\}) \rightarrow a$

- Type inference introduces "lacks" constraints on row variables

# Row Polymorphism

The idea of row (parametric) polymorphism is to use a type variable to represent any additional unknown fields:<sup>1</sup>

```
gcfBasket : (Num a, r/part/strike/cap/floor) =>
  (a, {part : a, cap : a, floor : a |r}) -> a
```

---

<sup>1</sup>Row polymorphism can be implemented with or without the lacks predicate, depending on whether repeated (scoped) labels are desired.

## Type constructors: Row kinds

- The empty row

$\langle \rangle : \text{ROW}$

- Extend a row type (one constructor per label):

$(\ell : \_ | \_) : \star \rightarrow \text{ROW} \rightarrow \text{ROW}$

## Type constructors: Records

- Construct a Record from a row type (gives product types, structurally):

$$\{_{\_}\} : \text{ROW} \rightarrow *$$

## Primitive operations on Records

- Selection

$$(\_) \cdot \ell : \forall ar. (r/\ell) \Rightarrow \{\ell : a \mid r\} \rightarrow a$$

- Restriction

$$(\_) / \ell : \forall ar. (r/\ell) \Rightarrow \{\ell : a \mid r\} \rightarrow \{r\}$$

- Extension<sup>2</sup>

$$\{\ell = \_ \mid \_\} : \forall ar. (r/\ell) \Rightarrow (a, \{r\}) \rightarrow \{\ell : a \mid r\}$$

---

<sup>2</sup>Note that Record literals are desugared to record extension.

## Enums and switching behaviour

```
function calcOffset(ccy)
  return
    if ccy == USD then 3
    else if ccy == JPY then 2
    else 0
end
```

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```
function calcOffset(ccy)
    return
        if ccy == USD then 3
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```

- No way to limit the set of atoms  
that can be used

## Enums and switching behaviour

```
function calcOffset(ccy)
    return
        if ccy == USD then 3
        else if ccy == JPY then 2
        else 0
    end
```

- No way to limit the set of atoms that can be used
- Forced to provide a default value in the else clause

## Enums and switching behaviour

```
function calcOffset(ccy)
  return
    case ccy of
      USD → 3,
      JPY → 2
end
```

## Enums and switching behaviour

```
function calcOffset(ccy)
  return
    case ccy of
      USD → 3,
      JPY → 2
end
```

## Row polymorphism

$\text{calcOffset} : \text{Num } a \Rightarrow \langle \text{USD}, \text{JPY} \rangle \rightarrow a$

## Enums and switching behaviour

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  return
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## Row polymorphism

$\text{calcOffset} : \text{Num } a \Rightarrow \langle \text{USD}, \text{JPY} \rangle \rightarrow a$

- Enums can be implemented using row types with unit fields

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end
```

## Row polymorphism

$\text{calcOffset} : \text{Num } a \Rightarrow \langle \text{USD}, \text{JPY} \rangle \rightarrow a$

- Enums can be implemented using row types with unit fields
- Note the top-level type is closed

## Extending enums and reusing behaviour

```
function calcOffsetExt(ccy)
    return
        case ccy of
            GBP → 3,
            otherwise c → calcOffset(c)
end
```

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```
function calcOffsetExt(ccy)
  return
    case ccy of
      GBP → 3,
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```

## Polymorphic extensible cases

$\text{calcOffset} : \text{Num } a \Rightarrow \langle \text{USD}, \text{JPY} \rangle \rightarrow a$

$\text{calcOffsetExt} : \text{Num } a \Rightarrow \langle \text{GBP}, \text{USD}, \text{JPY} \rangle \rightarrow a$

$c : \langle \text{USD}, \text{JPY} \rangle$

## Extending enums and reusing behaviour

```
function calcOffsetExt(ccy)
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end
```

## Polymorphic extensible cases

$\text{calcOffset} : \text{Num } a \Rightarrow$   
 $\langle \text{USD}, \text{JPY} \rangle \rightarrow a$

$\text{calcOffsetExt} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP}, \text{USD}, \text{JPY} \rangle \rightarrow a$

$c : \langle \text{USD}, \text{JPY} \rangle$

- Composition of cases using delegation

## Extending enums and reusing behaviour

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$c : \langle \text{USD}, \text{JPY} \rangle$

- Composition of cases using delegation
- Creates a new type containing a superset of fields

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$\text{calcOffsetExt} : \text{Num } a \Rightarrow \langle \text{GBP}, \text{USD}, \text{JPY} \rangle \rightarrow a$

$c : \langle \text{USD}, \text{JPY} \rangle$

- Composition of cases using delegation
- Creates a new type containing a superset of fields
- Flexibility similar to OOP subclassing, without giving up extensibility of functions

## Limiting the behaviour of existing code

```
function calcOffset2(ccy)
    return calcOffsetExt( <JPY |ccy> )
end
```

## Limiting the behaviour of existing code

```
function calcOffset2(ccy)
  return calcOffsetExt( ⟨JPY |ccy⟩ )
end
```

## Embedding

$\text{calcOffsetExt} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD, JPY} \rangle \rightarrow a$

$\text{calcOffset2} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD} \rangle \rightarrow a$

## Limiting the behaviour of existing code

```
function calcOffset2(ccy)
  return calcOffsetExt( ⟨JPY |ccy⟩ )
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## Embedding

$\text{calcOffsetExt} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD, JPY} \rangle \rightarrow a$

$\text{calcOffset2} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD} \rangle \rightarrow a$

- Embedding adds JPY to the type and restricts its values as possible input

## Overriding existing behaviour

```
function calcOffsetExt2(ccy)
    return
        case ccy of
            override USD → 4,
            otherwise c → calcOffsetExt(c)
end
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```
function calcOffsetExt2(ccy)
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      override USD → 4,
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end
```

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$\text{calcOffsetExt} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD, JPY} \rangle \rightarrow a$

$\text{calcOffsetExt2} : \text{Num } a \Rightarrow$   
 $\langle \text{GBP, USD, JPY} \rangle \rightarrow a$

$c : \langle \text{GBP, USD, JPY} \rangle$

- Override is syntactic sugar for embedding in the otherwise clause

## Optional arguments

```
function calcBasket(perfs, aggregation, weights)
    return case aggregation of
        Worst → minArray(perfs),
        Best → maxArray(perfs),
        Weighted
            → sumProduct(perfs, weights)
end
```

## Optional arguments

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            → sumProduct(perfs, weights)
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The weights argument is only  
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        Weighted(weights)
            → sumProduct(perfs, weights)
    end
```

The weights argument is only used in the Weighted case

## Variants

```
calcBasket : r/Worst/Best/Weighted ⇒
    (double[n]
     , ⟨Worst, Best, Weighted : double[n] | r⟩
    ) → double
```

Note that array sizes are also represented with a type variable

## Type constructors: Records and Variants

- Construct a record from a row type (gives product types, structurally):

$$\{_{\_}\} : \text{ROW} \rightarrow \star$$

- Construct a variant from a row type (gives sum types, structurally):

$$\langle_{\_}\rangle : \text{ROW} \rightarrow \star$$

## Primitive operations on Variants

- Injection (dual of selection)

$$\langle \ell = \_ \rangle : \forall ar. (r/\ell) \Rightarrow a \rightarrow \langle \ell : a \mid r \rangle$$

- Embedding (dual of restriction)

$$\langle \ell | \_ \rangle : \forall ar. (r/\ell) \Rightarrow \langle r \rangle \rightarrow \langle \ell : a \mid r \rangle$$

- Decomposition (dual of extension)

$$\langle \ell \in \_? \_ : \_ \rangle : \forall abr. (r/\ell) \Rightarrow \langle \ell : a \mid r \rangle \rightarrow a \oplus \langle r \rangle$$

## Primitive operations on Variants

- Injection (dual of selection)

$$\langle \ell = \_ \rangle : \forall ar. (r/\ell) \Rightarrow a \rightarrow \langle \ell : a \mid r \rangle$$
$$(\_.\ell) : \forall ar. (r/\ell) \Rightarrow \{\ell : a \mid r\} \rightarrow a$$

- Embedding (dual of restriction)

$$\langle \ell | \_ \rangle : \forall ar. (r/\ell) \Rightarrow \langle r \rangle \rightarrow \langle \ell : a \mid r \rangle$$
$$(\_ / \ell) : \forall ar. (r/\ell) \Rightarrow \{\ell : a \mid r\} \rightarrow \{r\}$$

- Decomposition (dual of extension)

$$\langle \ell \in \_? \_ : \_ \rangle : \forall abr. (r/\ell) \Rightarrow \langle \ell : a \mid r \rangle \rightarrow a \oplus \langle r \rangle$$
$$\{\ell = \_ | \_ \} : \forall ar. (r/\ell) \Rightarrow (a, \{r\}) \rightarrow \{\ell : a \mid r\}$$

- Decomposition (fused with a fold on the coproduct)<sup>3</sup>

`case _ of ℓ → _, otherwise → _ :`  
 $\forall abr. (r/\ell) \Rightarrow$   
 $(\langle \ell : a | r \rangle, a \rightarrow b, \langle r \rangle \rightarrow b) \rightarrow b$

- The empty alternative is used to close variants:

`emptyAlt : ⟨⟩ → b`

---

<sup>3</sup>Note that the case construct provides a notion of type refinement.

## Tracking Effects

```
function paySomething(amt, sched, settl)
  on sched pay Coupon amt with settl end
end
```

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function paySomething(amt, sched, settl)
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end
```

## Row-polymorphic effect types

`paySomething : (double, schedule, settlement) → {payments} ()`

- Use row types to add an effect parameter to every function

$$\forall a b. a \rightarrow \{e\} b$$

- Only consider effects that are intrinsic to the language.
- Assume strict semantics, a function call inherits the effects from evaluation of its arguments.

## “Lacks” constraint to restrict effects

We want to prevent users from making payments in case alternatives, as conditional payments must be handled via other primitives:

`case _ of ℓ → _, otherwise → _ :`

$\forall abre. (r/\ell, e/\text{payments}) \Rightarrow$

$(\langle \ell : a \mid r \rangle, a \rightarrow \{e\} b, \langle r \rangle \rightarrow b) \rightarrow b$

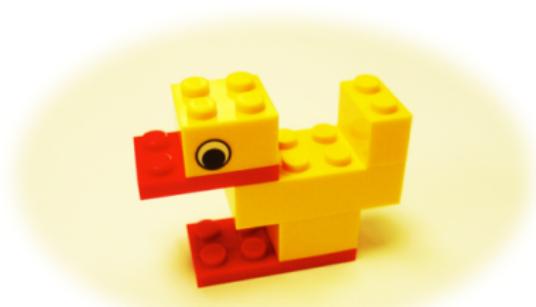
Note that we omit all unconstrained effect row variables.

## An example Lucid function type

```
autocallable : { asset : asset
    , fixedCouponAmt : double
    , asianInSchedule : schedule
    , asianOutSchedule : schedule
    , couponSchedule : schedule
    , autocallSchedule : schedule
    , digitalCouponParams : { direction : <Up, Down, StrictlyUp, StrictlyDown>
        , level : double
        , amount : double
    }
    , perfCouponOption : { type : <Call, Put, Forward, Straddle, Const>
        , strike : double
        , part : double
    }
    , autocallParams : { direction : <Up, Down, StrictlyUp, StrictlyDown>
        , level : double
        , amount : double
    }
    , maturityDate : schedule
    , finalRedemptionAmt : double
    , kiSchedule : schedule
    , kiBarrierParams : { direction : <Up, Down, StrictlyUp, StrictlyDown>
        , level : double
    }
    , kiOption : { type : <Call, Put, Forward, Straddle, Const>
        , strike : double
        , part : double
    }
} -> {payments, exit} ()
```

# Structural Typing

- Type equivalence determined by the type's actual structure, not by e.g. a name as in nominal typing.
- Research literature is almost completely concerned with structural type systems (TAPL 2002)



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- Permits sharing of constructors/labels amongst different types
- Allows reuse of code across different (extended) types
- No requirement or dependency on type definitions or declarations, types can be fully inferred from their usage and are self-describing
- Creation of a Nominal type from a Structural type, just requires “newtype” or similar. The inverse is not so easy.
- Combines the flexibility of untyping, with the safety of nominal typing
- Achieves the composition of data-types that we see in frameworks like Data types à la carte

## Structural Typing in Haskell

- Haskell vanilla ADTs and Records are nominally typed
  - We regularly see the tension of e.g. Tuples/HList versus Records.
  - But Haskell essentially uses structural typing exclusively for functions:
- 

Haskell	Java (Nominal)
$() \rightarrow \text{IO}()$	class Runnable { void run() }
$a \rightarrow a \rightarrow \text{Ordering}$	class Comparator { int compare(T, T) }
$a \rightarrow b$	class Function<? super T, ? extends R> { R apply(T) }

---

## An example type checker

<https://github.com/willtim/row-polymorphism>

# References

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